A Quiz III Solution, MA1025, Winter 2004

Note: I've resisted the temptation to combine steps. There are shorter proofs.

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Let A, B, and C be sets. Use the identity $D - E = D \bigcap E'$ and the laws of set operations to prove that

$$(A - C) - (B - C) = (A - B) - C.$$

Solution:

$$\begin{aligned}
(A-C)-(B-C) &= (A \bigcap C') \bigcap (B \bigcap C')' && \text{(The given identity, three times)} \\
&= (A \bigcap C') \bigcap (B' \bigcup C) && \text{(DeMorgan, Double Complement)} \\
&= ((A \bigcap C') \bigcap B') \bigcup ((A \bigcap C') \bigcap C) && \text{(Distributive Law)} \\
&= ((A \bigcap C') \bigcap B') \bigcup (A \bigcap (C' \bigcap C)) && \text{(Associative Law)} \\
&= ((A \bigcap C') \bigcap B') \bigcup (A \bigcap \emptyset) && (C \bigcap C' = \emptyset) \\
&= ((A \bigcap C') \bigcap B') \bigcup \emptyset && (A \bigcap \emptyset = \emptyset) \\
&= ((A \bigcap C') \bigcap B') && \text{(for any set } D, D \bigcup \emptyset = D) \\
&= ((A \bigcap B') \bigcap C') && \text{(Associative Law, Commutative Law)} \\
&= (A - B) - C && \text{(The initial identity, again).}
\end{aligned}$$